

Dynamic model to follow the state of charge of a lead-acid battery connected to photovoltaic panel

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ABSTRACT

The State of charge (SoC) of a lead-acid battery is linearly proportional to its open circuit voltage, which cannot be measured because the battery is continuously connected to the system that integrates it. This paper proposes an approach for estimating the open circuit voltage as a function of parameters of the electrical equivalent model. This approach is based on the recursive algorithm of Kalman Filter to estimate the State of charge of a battery dynamically. We also developed a state representation of a battery. Simulation and experimental validation of the model have been applied to a 'SLT210' lead-acid battery connected to a photovoltaic panel. Good concordance between estimation and measurement results is shown over periodic discharge, random discharge and charge/discharge cycles.

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1. Introduction

Electric energy is the most distributed form of energy. As well the use of electricity is clean, but its production processes have some level of environmental impact since it is generated by combusting fossil fuels. Exploitation of renewable resources such as solar, geothermal, and wind generally does not contribute to climate change or air pollution. The major drawback of renewable resources is their availability as they are intermittent and not permanent. To overcome this problem, storage devices like batteries is crucial to guarantee the load power supply in case of failure of renewable energy. Despite considerable improvements in its storage capacity and in its efficiency, the battery remains very sensitive to overloading and deep discharges. Such problems have direct influence on battery life period. A precise knowledge of the battery's state of charge (SoC) is necessary to preserve it. Indeed, good management of the stored electrical energy leads to an optimization of consumption and guarantees the safety of the battery. However, the SoC is a non-measurable quantity but it can be deduced while knowing the open circuit voltage of the battery (V_{oc}). This quantity cannot be measured since the battery cannot be disconnected from the system.

The state of charge is a fundamental parameter for any application using the battery. The knowledge of this parameter ensures the effective functioning and the security of the battery. In addition, the strategy of energy management is given through the exact estimation of SoC. Generally, this estimation depends on many factors such as temperature, capacity and internal resistance of the battery.

Various techniques have been proposed to determine the SoC of a lead-acid battery. Coulomb-metric estimation is commonly used given its simplicity [1]. However, this method presents limitation because of its reliance on integration, errors in current measurement due to noise, cumulative resolution and rounding. The estimation by impedance measurement approach consists in imposing a voltage or a current excitation to the battery. The internal impedance of the battery will be deduced from its response [2]. As it is based on measurements, this method is pragmatic. Nevertheless, the SoC calculation requires the disconnection of the battery from the installation. Hence, it is considered an offline SoC calculation method. The same limitation is presented by another offline method which consists of a linear approximation between measured V_{oc} and SoC [3]. Non conventional methods as Artificial Neural Network can overcome this limitation. This approach estimates the SoC on the basis of a measurements learning database which requires a continuous update [4]. Estimation by Kalman Filter (KF) consists of finding the best estimation of the real-time status based on carried out observations. This technique is considered the most accurate one compared to previously presented approaches. In fact, it is based on a correction mechanism of the prediction errors [5,6].

This paper suggests an approach to dynamically determine the SoC of a load connected battery. First, the capacitors terminal voltages of the battery equivalent electrical circuit are estimated by means of a Kalman Filter (KF). Then, the battery SoC is calculated following the determination of the open circuit voltage which is expressed as a function of estimated capacitors terminal voltages. The KF estimation is accomplished thanks to the establishment of the battery state model. The algorithm has been validated by conducting experiments on a battery connected to a photovoltaic panel (PVP) that supplies a resistive load.

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2. Plant presentation

The considered plant is composed of a photovoltaic panel (SM50: Siemens) connected to a lead acid battery (SLT210) and a resistive load via a charge/discharge controller (Fig. 1). The PVP can be used to supply the load or to charge the battery in case of low SoC (battery discharged). When PVP power is unavailable, the battery ensures required load supply.

The current I_{pv} , delivered by the PVP, is a function of the global solar radiation (G) and the ambient temperature (T_a). This current is expressed by Eq. (1) [7]:

$$I_{pv} = 3.33 + 1.4 \times 10^{-3} \left(\frac{G}{1000} \right) \left(T_a + \frac{G}{40} - 25 \right) + 3.35 \left(\frac{G}{1000} - 1 \right) \quad (1)$$

3. Battery model

The operation of a battery can be described in two different ways: by a series of empirical chemical equations based on the observation or by an electrical circuit reproducing the electrical quantities evolution. Various electric equivalent circuit have been applied to lead-acid batteries to determine the SoC. However, the complex nonlinear electrochemical processes that occur during power transfer to/from the battery, requires a complex electrical model to accurately describe the SoC in a dynamic way [6].

Fig. 2 presents the equivalent circuit of a lead acid battery which is connected to a photovoltaic panel and a load. It is composed of a resistance R_t in series with two parallel branches. The first branch consists of a capacitor C_1 that characterizes the storage capacity of the battery, in series with a resistance R_1 . The second branch includes a capacitor C_2 that reflects the effects of diffusion within the battery, in series with a resistance R_2 [6,9].

By applying the law of meshes on the equivalent circuit of the battery, the battery voltage is expressed as:

$$V_t(t) = I(t) \cdot R_t + I_1(t) \cdot R_1 + V_{C1}(t) \quad (2)$$

$$V_t(t) = I(t) \cdot R_t + I_2(t) \cdot R_2 + V_{C2}(t) \quad (3)$$

This leads to:

$$I_1(t) \cdot R_1 = I_2(t) \cdot R_2 + V_{C2}(t) - V_{C1}(t) \quad (4)$$

Since $I(t) = I_2(t) + I_1(t)$, Eq. (4) becomes:

$$I_1(t) \cdot (R_1 + R_2) = I(t) \cdot R_2 + V_{C2}(t) - V_{C1}(t) \quad (5)$$

Let $V_t = V_t(t)$, $V_{C1} = V_{C1}(t)$, $V_{C2} = V_{C2}(t)$ and $I = I(t)$, $I_1 = I_1(t)$, $I_2 = I_2(t)$

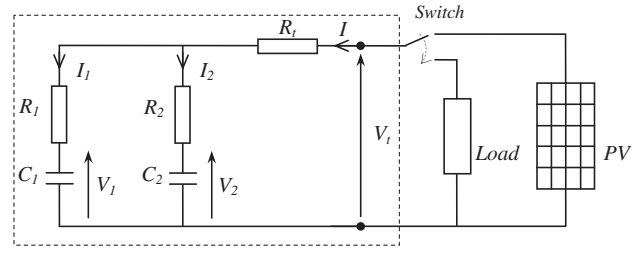


Fig. 2. Equivalent electrical circuit of the battery.

The current through a capacitor is proportional to the derivative of the voltage across its terminals:

$$I_1 = \dot{V}_{C1} \cdot C_1 \quad (6)$$

Eqs. (5) and (6) make possible to determine the expression of $\dot{V}_{C1}$ function of the different parameters of the model, as follows:

$$\dot{V}_{C1} = -\frac{V_{C1}}{C_1(R_1 + R_2)} + \frac{V_{C2}}{C_1(R_1 + R_2)} + \frac{R_2}{C_1(R_1 + R_2)} I \quad (7)$$

Similarly:

$$\dot{V}_{C2} = -\frac{V_{C2}}{C_2(R_1 + R_2)} + \frac{V_{C1}}{C_2(R_1 + R_2)} + \frac{R_1}{C_2(R_1 + R_2)} I \quad (8)$$

Using Eqs. (2) and (5) the terminal voltage of the battery V_t is equal to:

$$V_t = \left[\frac{R_2}{R_1 + R_2} \quad \frac{R_1}{R_1 + R_2} \right] \begin{bmatrix} V_{C1} \\ V_{C2} \end{bmatrix} + \left[R_t + \frac{R_1 R_2}{R_1 + R_2} \right] I \quad (9)$$

Assuming that the rate (dI/dt) is negligible, the derivative of V_t is given by:

$$\dot{V}_t = \left[-\frac{R_2}{C_1(R_1 + R_2)^2} + \frac{R_1}{C_2(R_1 + R_2)^2} \right] \cdot V_{C1} + \left[\frac{R_2}{C_1(R_1 + R_2)^2} - \frac{R_1}{C_2(R_1 + R_2)^2} \right] \cdot V_{C2} + \left[\frac{R_2^2}{C_1(R_1 + R_2)^2} + \frac{R_1^2}{C_2(R_1 + R_2)^2} \right] \cdot I \quad (10)$$

This can be expressed as a standard state-space model:

$$\begin{cases} \dot{X}(t) = AX(t) + BI(t) \\ V_t(t) = CX(t) + DI(t) \end{cases}$$

For this, Eqs. (7), (8), and (10) yield the following state-space representation:

$$\begin{cases} \begin{bmatrix} \dot{V}_{C1} \\ \dot{V}_{C2} \\ \dot{V}_t \end{bmatrix} = \begin{bmatrix} -\frac{1}{C_1(R_1 + R_2)} & \frac{1}{C_1(R_1 + R_2)} & 0 \\ \frac{1}{C_2(R_1 + R_2)} & -\frac{1}{C_2(R_1 + R_2)} & 0 \\ A(3,1) & 0 & A(3,3) \end{bmatrix} \begin{bmatrix} V_{C1} \\ V_{C2} \\ V_t \end{bmatrix} \\ + \begin{bmatrix} \frac{R_2}{C_1(R_1 + R_2)} \\ \frac{R_1}{C_2(R_1 + R_2)} \\ B(3,1) \end{bmatrix} I \quad Y = [001] \begin{bmatrix} V_{C1} \\ V_{C2} \\ V_t \end{bmatrix} + 0 \cdot I. \end{cases}$$

Let:

$$A(3,1) = -R_2/(C_1(R_1 + R_2)^2) + R_1/(C_2(R_1 + R_2)^2) - (R_2^2)/(C_1 R_1 (R_1 + R_2)^2) + R_2/(C_2(R_1 + R_2)^2)$$

$$A(3,3) = \frac{R_2}{C_1 \cdot R_1 (R_1 + R_2)} - \frac{1}{C_2(R_1 + R_2)}$$

$$B(3,1) = \frac{R_1^2}{C_2(R_1 + R_2)^2} - \frac{R_2 R_t}{C_1(R_1 + R_2)} + \frac{R_t}{C_2(R_1 + R_2)} + \frac{R_1 R_2}{C_2(R_1 + R_2)^2}$$

$X(t) = \begin{bmatrix} V_{C1}(t) \\ V_{C2}(t) \\ V_t(t) \end{bmatrix}$: The State vector, $V_t(t)$ is the output and $I(t)$ is the input.

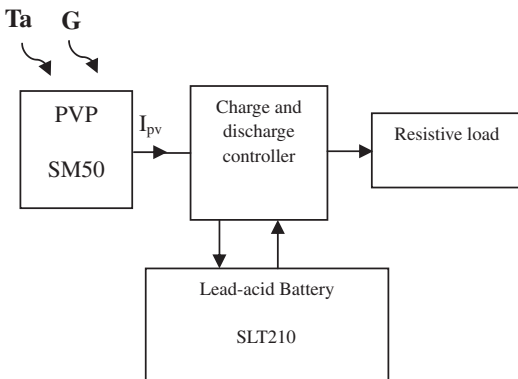


Fig. 1. Photovoltaic/battery/load plant.

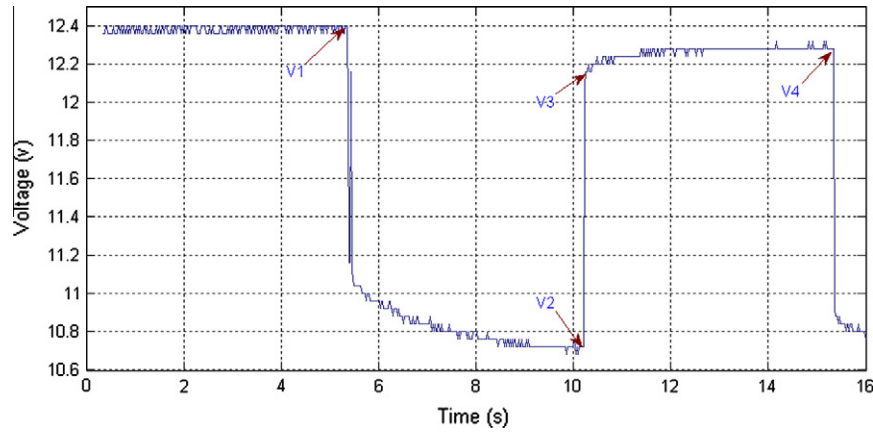


Fig. 3. The terminal voltage of the battery for a rapid discharge cycle.

Table 1
Parameters of the battery model.

Parameter	R	R_t	R_1	R_2	C_1	C_2
Value	.021	.0126	.0168	.0168	284693.87	103.02

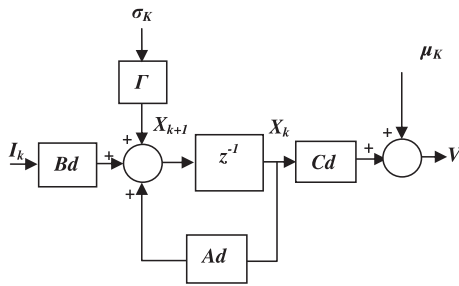


Fig. 4. Block diagram of battery model.

4. Identification of the battery model parameters

The model parameters of the lead-acid battery are determined from experimental tests. In the context of this work, the solar battery is of type ASSAD SLT210. This battery is a monobloc consisting of six cells connected in series. Each cell is formed by a tubular type positive electrode and a planar type negative electrode, both immersed in a diluted sulfuric acid electrolyte.

4.1. Battery resistance

When the battery is connected to the charge, the start value of the terminal voltage and the current are carefully saved. The internal resistance of the battery R is expressed as a function of these values and the vacuum voltage value E (Eq. (11)) [6].

$$R = \frac{E - V_t}{I} \quad (11)$$

On the other hand, the internal resistance of the battery is equal to the equivalent resistance deduced from the electrical equivalent circuit of the battery:

$$R = R_t + \frac{R_1 \cdot R_2}{R_1 + R_2} \quad (12)$$

Assuming that R_1 and R_2 are equivalent and represent 80% of the total resistance gives:

$$R_1 = R_2 = 0.8R \quad (13)$$

$$R_t = 0.6R \quad (14)$$

4.2. Capacitor C_1

The value of the capacity C_1 is calculated from the energy E_{C1} stored in the battery. This energy is expressed as a function of V_{oc} at 20% and 100% of SoC by the following expression [8,9]:

$$E_{C1} = \frac{1}{2} C_1 (V_{oc(100\%SoC)}^2 - V_{oc(20\%SoC)}^2) \quad (15)$$

where $V_{oc(100\%SoC)}$ and $V_{oc(20\%SoC)}$ are respectively the initial and the minimum open circuit voltage. Both of them are provided by the manufacturer. On the other hand, the energy E_{C1} is equivalent to the capacity of the battery in ampere-second:

$$E_{C1} = 3600 \times C20 \times V_{oc(100\%SoC)} \quad (16)$$

where $C20$ is the ability of the battery for a discharge current $I = 9$ A during 20 h which yields $C20 = 180$ AH.

Therefore, the value of the capacity C_1 is equal to:

$$C_1 = \frac{3600 \cdot C20 \cdot V_{oc(100\%SoC)}}{\frac{1}{2} C_1 (V_{oc(100\%SoC)}^2 - V_{oc(20\%SoC)}^2)} \quad (17)$$

4.3. Capacitor C_2

The initial value of the capacity C_2 is calculated from the time constant τ which is determined by discharging at very important current during a very small time interval. It unloads the battery by a high current of 30 A for a period of 5 s. The curve of the voltage for this experiment is presented by Fig. 3:

The time constant τ can be expressed by the following relation [8]:

$$V_4 = V_3 + (V_1 - V_3)(1 - e^{-t/\tau}) \quad (18)$$

Therefore:

$$\tau = -\Delta t \ln \left(1 - \frac{V_4 - V_3}{V_1 - V_3} \right) \quad (19)$$

$$\tau = -5 * \ln \left(1 - \frac{12.28 - 12.08}{12.36 - 12.08} \right) = 6.26 \text{ s}$$

In addition, the time constant τ can be expressed as a function of the capacity C_2 and its associated resistors while eliminating the effects of the capacity C_1 [11].

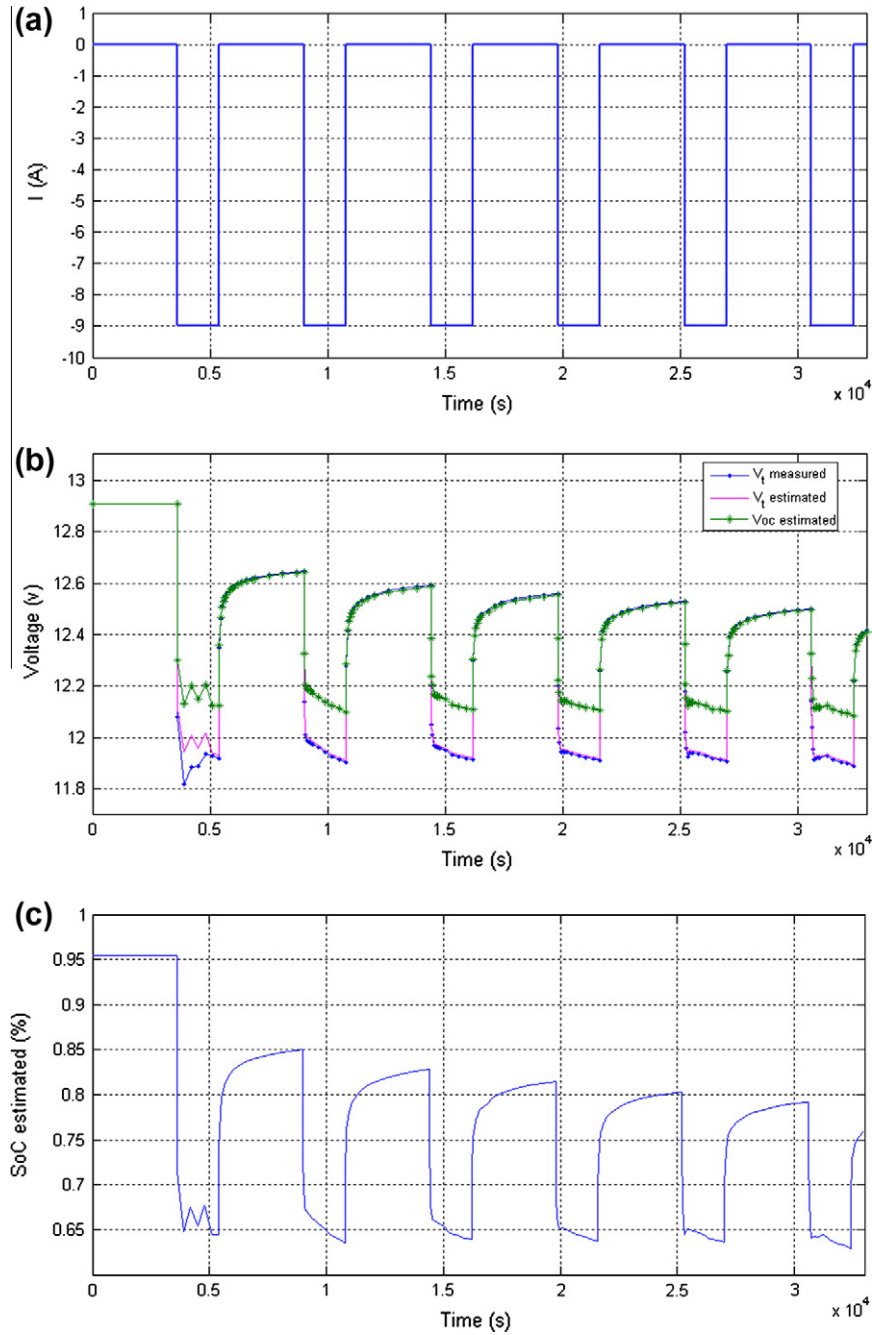


Fig. 5. Dynamic behaviors of the battery model in response to discharge pulses of 9A. (a) Discharge current. (b) Measured and estimated V_t and the estimated $\hat{V}_{oc}$. (c) Estimated SoC.

$$\tau = (R_1 + R_2)C_2 \quad (20)$$

R_1 and R_2 are given by Eq. (13).

Hence the value of the capacity C_2 is:

$$C_2 = \frac{\tau}{R_1 + R_2} \quad (21)$$

Table 1, summarizes the identified parameters of the battery model.

5. Estimation of the State of charge by Kalman Filter

The technique of estimating SoC by KF is based on a dynamic modeling of battery by a discrete state representation (22) taking into account the input noise and measure noise [10]. Discrete state

model is obtained from the continuous model considering that the current I is constant during each sampling period.

$$\begin{cases} X_k = A_d X_{k-1} + B_d I_{k-1} \\ V_{t_k} = C_d X_k \end{cases} \quad (22)$$

where:

$$A_d = Id_3 + A \cdot T_c$$

$$B_d = B \cdot T_c$$

$$C_d = C$$

A , B , and C are the battery model matrices, T_c is the sampling period, and Id_3 the identity matrix. The system is now assumed to be

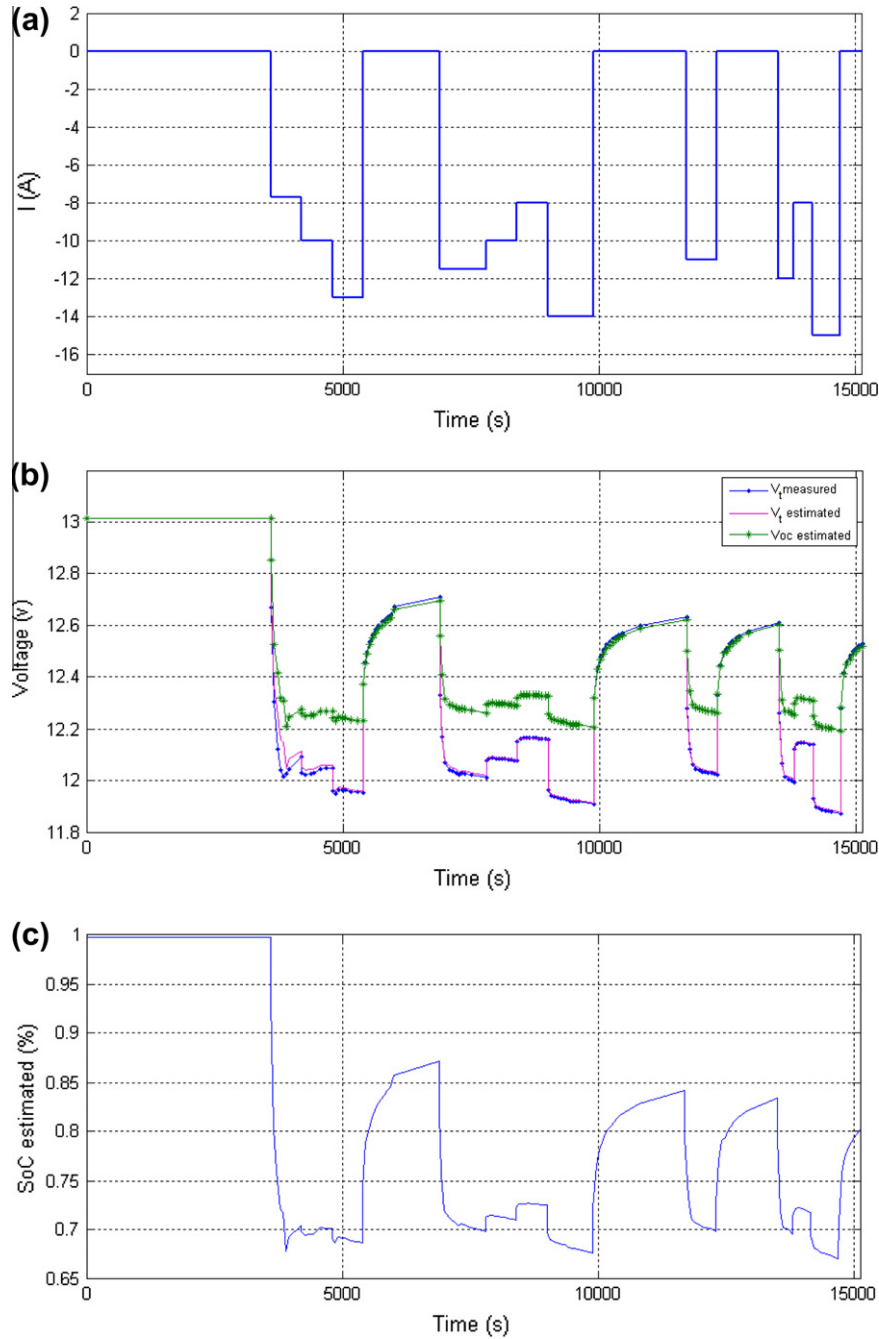


Fig. 6. Dynamic behaviors of the battery model in response to random discharge. (a) Discharge current. (b) Measured and estimated V_t and the estimated $\hat{V}_{oc}$. (c) Estimated SoC.

corrupted by stationary Gaussian white noise, σ_K being used to represent system disturbances and model inaccuracies μ_k and represents the effects of measurement noise. The state representation becomes:

$$\begin{cases} X_k = A_d X_{k-1} + B_d I_{k-1} + \Gamma \cdot \sigma_{k-1} \\ V_{t_k} = C_d X_k + \mu_k \end{cases} \quad (23)$$

The noises σ_K and μ_k have a zero mean value and a covariance respectively denoted Q and R .

$$E[\sigma_k \sigma_k^T] = Q \quad (24)$$

$$E[\mu_k \sigma_k^T] = R \quad (25)$$

A schematic of the state representation of the battery is given in Fig. 4.

The KF can provide the best possible estimation of the state vector X_k , based on observations up to a given moment, within the meaning of minimizing the criterion the variance as follows:

$$\text{Min}\{E[(X_k - \hat{X}_{k/k}) \cdot (X_k - \hat{X}_{k/k})^T]\} \quad (26)$$

The algorithm of the KF involves two steps [10,11]: a prediction step and a correction step. The prediction step is based on an estimation of the state X_k at the moment k , given by Eq. (27), knowing all values of the output measured up to the moment $k - 1$. In this step the covariance matrix of error is calculated so that it can be used in the correction step to optimize the gain of the filter (Eq. (28)).

- Predicted state vector

$$\hat{X}_{k/k-1} = A_d \hat{X}_{k-1/k-1} + B_d I_{k-1} \quad (27)$$

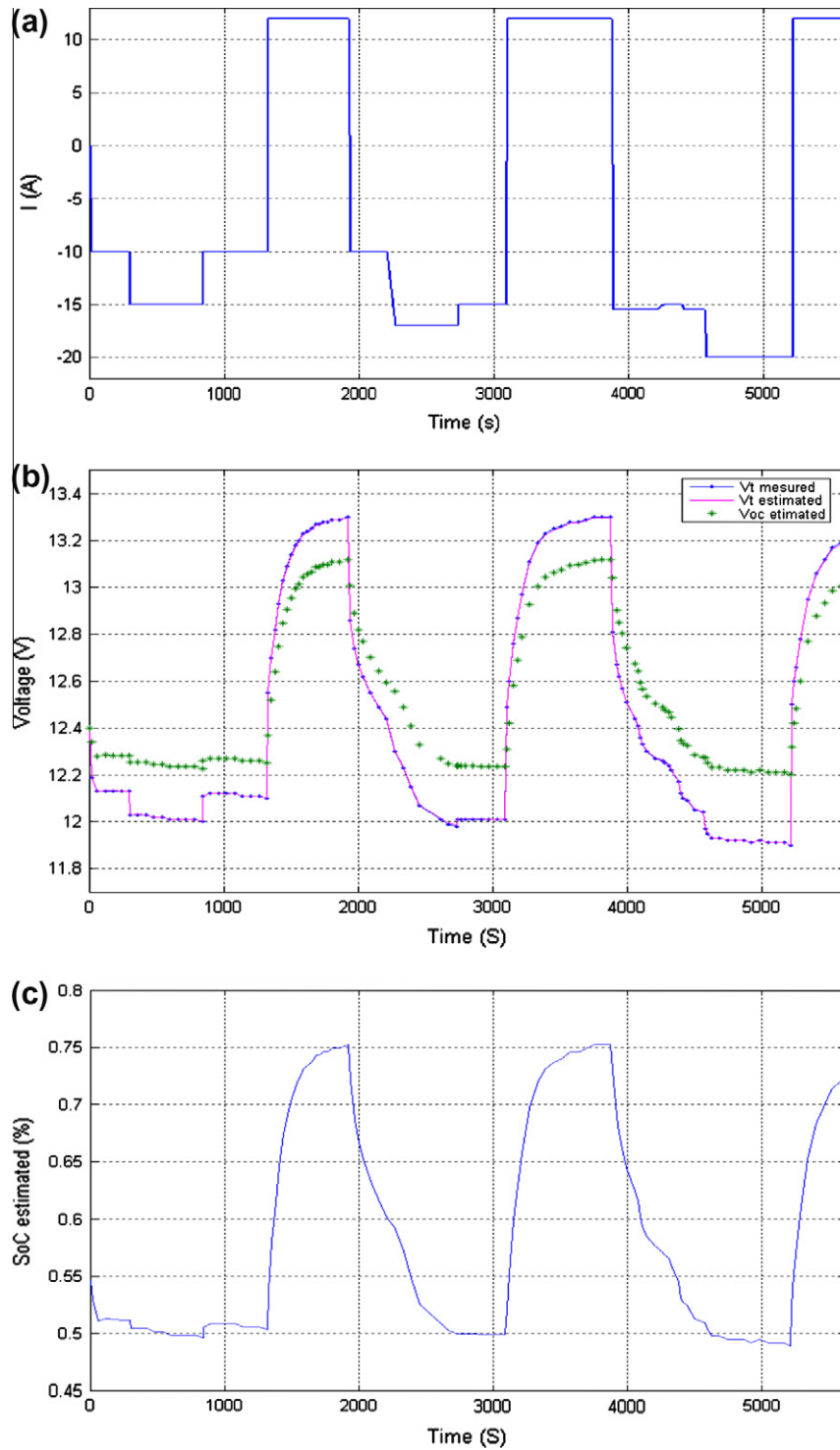


Fig. 7. Dynamic behaviors of the battery model in response to charge/discharge cycle. (a) Charge/discharge current. (b) Measured and estimated V_t and the estimated $\hat{V}_{oc}$. (c) Estimated SoC.

- Prediction error covariance

$$P_{k/k-1} = A_d \cdot P_{k-1/k-1} \cdot A_d^T + \Gamma Q \Gamma^T \quad (28)$$

The correction step minimizes the estimation error calculated by the gain of the filter. The predicted state will be corrected following the new measured value of the output.

- Gain filter

$$K_k = P_{k/k-1} \cdot C_d^T [C_d P_{k/k-1} C_d^T + R]^{-1} \quad (29)$$

- Correction of the prediction

$$\hat{X}_{k/k} = \hat{X}_{k/k-1} + K_k \cdot (Y_k - C_d \cdot \hat{X}_{k/k-1}) \quad (30)$$

- Error of covariance

$$P_{k/k} = (I - K_k \cdot C_d) \cdot P_{k/k-1} \quad (31)$$

Given that only the input I and the output V_t of the battery can be measured, these two variables are assumed invariant during the time state estimation. The initialization of the covariance matrices of noise input Q , of measurements R and of error P is made before the first iteration of calculation of the filter.

Assume:

$$\text{Input covariance matrix : } Q = \begin{bmatrix} 0.008 & 0 & 0 \\ 0 & 0.08 & 0 \\ 0 & 0 & 0.8 \end{bmatrix}$$

Measurements covariance matrix: $R = 1$

$$\text{Error covariance matrix : } P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The estimated open circuit voltage $\hat{V}_{oc}$ is expressed by [6]:

$$\hat{V}_{oc} = n(E_1 + \text{SoC} \cdot (E_0 - E_1)) \quad (32)$$

where n is the number of cells in the battery, $E_0 = 2.17v$ is the unloaded voltage of a cell, $E_1 = 1.75v$ is the limit discharge voltage of a cell.

In addition, the $\hat{V}_{oc}$ can be expressed in terms of voltage at the terminal capacitors C_1 and C_2 as:

$$\hat{V}_{oc} = \left[\frac{R_2}{R_1 + R_2} \frac{R_1}{R_1 + R_2} \right] \begin{bmatrix} \hat{V}_{C1} \\ \hat{V}_{C2} \end{bmatrix} \quad (33)$$

Hence, the estimated SoC is given by:

$$\text{SoC} = \frac{1}{E_0 - E_1} \left[\frac{R_2}{R_1 + R_2} \frac{R_1}{R_1 + R_2} \right] \begin{bmatrix} \hat{V}_{C1} \\ \hat{V}_{C2} \end{bmatrix} - \frac{n \cdot E_1}{E_0 - E_1} \quad (35)$$

6. Experimental validation

To validate the theoretical results, three experimental tests were carried out on a *SLT210* lead-acid battery. The first is to discharge the battery periodically, the second consists of a random discharge and the third is a charge and discharge cycle.

6.1. Periodic discharge

To follow the dynamic behavior of the battery, a periodic operating cycle is applied by a discharge current. This cycle consists of a discharge phase for half an hour and a pause phase during 1 h (Fig. 5a). The first step of estimation is to determine the value of the terminal voltage of the battery. Good concordance between the measured and estimated values is shown in Fig. 5b. In addition, this figure represents the curves of the $\hat{V}_{oc}$. It is clear that during the discharge, the value of $\hat{V}_{oc}$ is higher than the value of $\hat{V}_t$. For the no-load operations the two voltages are similar. Consequently, by applying Eq. (35), the estimated behavior of SoC is shown in Fig. 5c.

6.2. Random discharge

For this test, a non periodic operating cycle was chosen to be closest to real operating conditions. The current delivered by the battery during discharge and the pause is illustrated in Fig. 6a.

The evolutions of $\hat{V}_t$, V_t and $\hat{V}_{oc}$ are shown in Fig. 6b. Note that even for a non-periodic discharge, the estimated voltage follows the measured voltage.

In addition, Fig. 6c gives the evolution of the SoC which is computed using Eq. (35). Similarly to what is seen in periodic discharge, the evolution of the SoC follows the load curve.

6.3. Charge and discharge cycle

A charge and discharge cycle is applied to a battery connected to a PVP and a resistive load (Fig. 7a). Fig. 7b shows that the estimated voltage follows the measured voltage. The evolution of the SoC, which is computed using Eq. (35), is given in Fig. 7c.

During charge/discharge phases, the values of measured and estimated voltages (V_t) show a good concordance. Consequently, V_{oc} and SoC show the same behavior as V_t .

7. Conclusion

As a connected battery state of charge cannot be measured, an estimation algorithm for this parameter based on a Kalman Filter (KF) is presented. The algorithm forecasts SoC following the measure of the battery voltage under load. First, a battery state model is established. The model parameters identification is ensured thanks to measurements driven on the battery. Second, a KF algorithm has been developed in order to estimate the voltage V_{C1} and V_{C2} of the equivalent electrical circuit and subsequently to estimate the open circuit voltage while the battery is connected to a load. The estimator is based on the state model and the measured load battery voltage. The estimator has been validated on a *SLT210* solar battery over periodic discharge, random discharge and charge/discharge cycle. The developed estimator reflects the dynamic behavior of the battery following a great concordance between estimated and measured battery voltages. Finally, the estimated open circuit voltage V_{oc} is used to determine the state of charge (SoC).

In a future work, the estimated SoC will be used to define an optimum control for the battery charging which must offer better safety and life duration for the battery.

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